of Aircraft, Vol. 29, No. 3, 1992, pp. 458–464.

[4]Qu, B., "Joined Wing Model Vibrations Using PC-Based Modal Testing and Finite Element Analysis," M.S. Thesis, Graduate School, Auburn Univ., Auburn, AL, Dec. 13, 1991.

[5]*STAR Reference Manual*, Structural Measurement Systems, Milpitas, CA, Jan. 1990.

[6]*WAVEPAK User's Manual*, Computational Systems Inc., Knoxville, TN, Aug. 1989.

[7]Gockel, M. A. (ed.), *MSC/NASTRAN—Handbook for Dynamic Analysis*, Version 63, The MacNeal-Schwendler Corp., Los Angeles, CA, June 1983.

[8]Qu, B., and Cutchins, M. A., "Joined Wing Model Vibrations Using PC-Based Modal Testing and Finite Element Analysis," *Proceedings of the AIAA/ASME/ASCE/AHS/ASC 33rd Structures, Structural Dynamics, and Materials Conference* (Dallas, TX), AIAA, Washington, DC, 1992, pp. 1407–1413 (AIAA Paper 92-2260).

Parameter Estimates of an Aeroelastic Aircraft as Affected by Model Simplifications

A. K. Ghosh* and S. C. Raisinghani†
Indian Institute of Technology, Kanpur 208016, India

Introduction

PARAMETER estimation from flight data as applied to aircraft in the linear flight regime is currently being used on a routine basis.[1] However, the rigid body models used successfully up to this time seem to be inadequate for newly introduced highly maneuverable aircraft with a high degree of structural flexibility. Zerweckh et al.[2] have suggested that research is required to include appropriate aeroelastic models in parameter estimation algorithms for flexible aircraft. Waszak and Schmidt[3] suggested a simplified integrated modeling approach to account for aeroelastic effects in aircraft dynamics, and its use for highly elastic aircraft was demonstrated in Ref. 3. Furthermore, several model reduction methods have been considered in Ref. 3 and the frequency responses of the resulting models (full order, rigid, residualized, and truncated) are compared to show how mismatch among them increases as the flexibility of the aircraft is increased.

A full order model of an aeroelastic aircraft has too many parameters to yield satisfactory estimates by using any of the known parameter estimation methods.[4] In view of this, a preliminary study has been initiated in this Note to investigate how estimation model may be simplified to reduce the number of unknown parameters, and how are the resulting parameter estimates affected by such approximate models. Special attention has been paid to the extreme case of using rigid body model in the estimation algorithm, and an analytical method is proposed to predict approximations to parameter estimates expected from the use of rigid body models.

Methodology

Only the longitudinal axis nonlinear equations of motion of a flexible aircraft given in Ref. 3 are considered, and linearized about a straight and level cruise flight. Assuming variations in velocity to be negligible (u = constant), the equations of motion of Ref. 3 may be approximated for the short period mode as

$$\dot{\alpha} - q = \rho u S/2m \left[C_{z_\alpha}\alpha + C_{z_q} qc/2u + C_{z_\delta}\delta + \sum_{i=1}^{n} (C_{z_{ni}} n_i + C_{z_{\dot{n}i}} \dot{n}_i c/2u) \right] \tag{1a}$$

$$\dot{q} = \rho u^2 Sc/2I_y \left[C_{m_\alpha}\alpha + C_{m_q} qc/2u + C_{m_\delta}\delta + \sum_{i=1}^{n} (C_{m_{ni}} n_i + C_{m_{\dot{n}i}} \dot{n}_i c/2u) \right] \tag{1b}$$

where vehicle angle of attack α, pitch rate q, and control input δ represent small perturbations from the chosen reference flight conditions. n_i and $\dot{n}_i$ are the generalized elastic deflections and their time derivatives. C_{z_α}, C_{m_δ} . . . are the stability and control derivatives as defined in Ref. 3. Air density ρ, total inertial velocity u, wing area S, wing chord c, aircraft mass m, and moment of inertia about Y axis I_y are the other quantities used in the above equations.

The following equation satisfied by the generalized coordinates n_i is taken from Ref. 3, except that a term $2\xi_i\omega_i\dot{n}_i$ representing the structural damping is also included into it

$$\ddot{n}_i + 2\xi_i\omega_i\dot{n}_i + \omega_i^2 n_i = \frac{\rho u^2 Sc}{2M_i} \left[C_\alpha^{ni}\alpha + C_q^{ni} qc/2u + C_\delta^{ni}\delta + \sum_{j=1}^{n} (C_{nj}^{ni} n_j + C_{\dot{n}j}^{ni} \dot{n}_j c/2u) \right] \tag{2}$$

where ω_i, ξ_i, and M_i are the in-vacuo frequency, modal damping, and modal generalized mass, respectively. The total force coefficients have the usual meaning as given in Ref. 3.

Rigid Body Modeling

The simplest possible approach to parameter estimation from flight data of an aeroelastic aircraft is to use a rigid body estimation model. Since in actual flight data, the effects of all the aeroelastic modes will necessarily be present, it is expected that the aeroelastic effects will get absorbed into the estimated parameters using rigid body model in estimation algorithm. For convenience, we shall refer to such parameter estimates by the name "equivalent parameters."

The basic question posed is, when and how useful are these estimated equivalent parameters. Can we analytically predict the expected values of equivalent parameters. For this purpose, it is assumed that the contributions of the generalized elastic deflections (n_i) are instantaneous. Steady-state expression for $(n_i)_{ss}$ is obtained from Eq. (2) by dropping the time derivative terms.

$$(n_i)_{ss} = \rho u^2 Sc/2M_i\omega_i^2 \left[C_\alpha^{ni}\alpha + C_q^{ni} qc/2u + C_\delta^{ni}\delta + \left(\sum_{j=1}^{n} C_{nj}^{ni} n_j + C_{\dot{n}j}^{ni} \dot{n}_j c/2u \right) \right] \tag{3}$$

Next, Eq. (3) is substituted into Eq. (1). Collecting the coefficients of various flight variables and control input yield approximate analytical expressions for the equivalent parameters. These represent approximations to the equivalent de-

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*Graduate Student; currently Scientist, Armament Research and Development Establishment, Pune 411021, India

†Professor, Aerospace Engineering Department, P.O. I.I.T.

rivatives because of the assumption made in arriving at Eq. (3). Since the analytical expressions for equivalent parameters are quite lengthy when expressions for more than one mode ($i \geq 2$) from Eq. (3) are substituted into Eq. (1), we illustrate the form of the equivalent parameters obtained corresponding to absorption of only the first mode. Substitution of Eq. (3) into Eq. (1) for $i = 1$, and collecting coefficients of α, q, and δ, we obtain analytical expressions for the equivalent parameters as

$$C'_{z_x} = C_{z_x} + FC_{z_{n1}}C_x^{n1}; \qquad C'_{m_x} = C_{m_x} + FC_{m_{n1}}C_x^{n1}$$
$$x = \alpha, q \quad \text{or} \quad \delta$$

where

$$F = (2M_1\omega_1^2/\rho u^2 Sc - C_{n1}^{n1})^{-1} \tag{4}$$

Eq. (4) shows how the equivalent parameters C'_{z_x} and C'_{m_x} can be analytically computed. The second terms on the right side of these equations show the manner in which aeroelastic effects get absorbed with the rigid body parameters. Therefore, the difference between the equivalent and rigid body parameters is given by these second terms on the right side of Eq. (4), and the difference is proportional to the numerical value of the function F. As may be seen from the expression for F, it varies inversely with the in-vacuo frequency ω. An increase in the flexibility of aircraft implies decrease in ω, and therefore, an increase in the numerical value of F, resulting in a larger difference between the equivalent and rigid body parameters. Furthermore, it may be noted that for a perfectly rigid aircraft ($\omega \rightarrow \infty$), the numerical value of F tends to zero, and thus the equivalent parameters become, as they must, the same as that for a rigid aircraft.

Simulated Flight Data

Since the real flight data for a flexible aircraft could not be obtained, the present study has been carried out with simulated data for an example aircraft similar to that given in Ref. 3. Using the geometric, mass, moment of inertia characteristics, the stability and control derivatives, the flight condition, and the first four aeroelastic modes given for the baseline configuration C2 in Ref. 3, frequency responses for the longitudinal short period mode were generated. The responses of interest are α/δ, q/δ, and a_z/δ where a_z is the normal acceleration defined by

$$a_z = u/g(\dot{\alpha} - q) \tag{5}$$

To vary the flexibility of the aircraft, it is assumed that aircraft can be made to vibrate in nearly the same normal modes but with different in-vacuo frequencies.[5] Thus, for more flexible aircraft C3 of Ref. 3, the same total force coefficients as that of C2—but in-vacuo frequencies of C3—were used to generate its frequency responses.

Parameter Extraction

The parameter estimation technique employed was maximum likelihood method in the frequency domain.[6] The computer generated responses always had the aeroelastic effects included for all the four modes, while the estimation model used in the estimating algorithm was increasingly simplified to see its effect on the accuracy of parameter estimates. To this purpose, results are presented for four cases wherein the model included or excluded some or all of the aeroelastic modes: case 1—all the four modes included; case 2—only the first and the second modes included; case 3—only the first mode included; and case 4—none of the modes included (rigid body modeling).

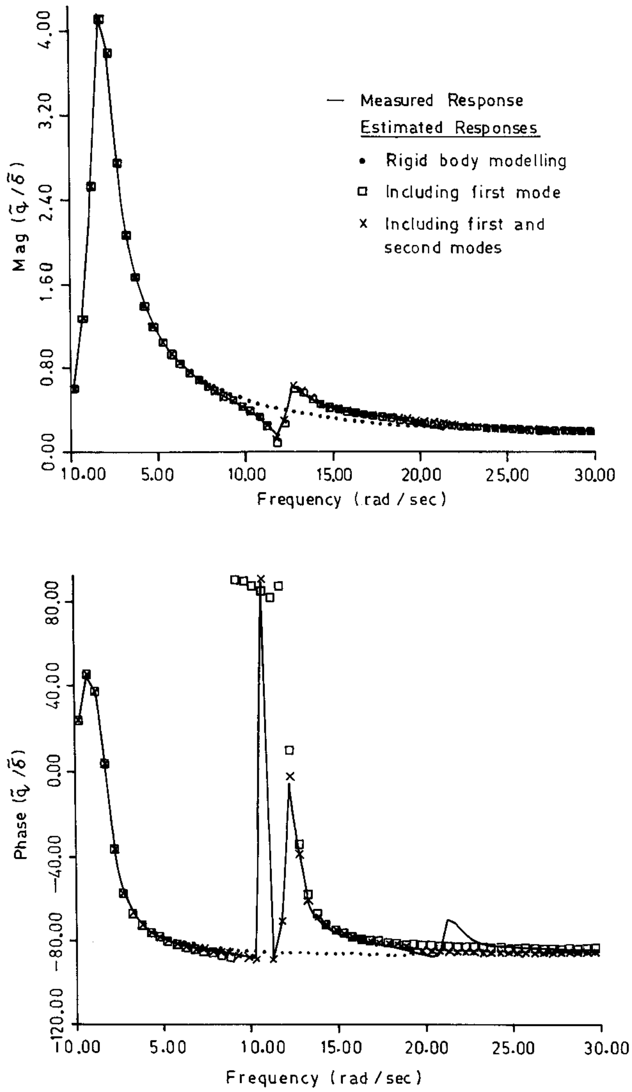

Fig. 1 Comparison of frequency responses for cases 1, 2, 3, and 4.

For case 1, as expected for simulated data, the estimated values compared almost exactly with the true values. The estimated and true responses for case 1 show almost perfect matching and are indistinguishable on the scale of Fig. 1.

For cases 2 and 3, the estimated parameters are shown in the 4th and 5th columns of Table 1. It is observed that the inclusion of only the first mode (case 3) results in estimated parameters C_{z_α}, C_{m_α}, C_{m_q}, and C_{m_δ} being in good agreement with the true values while C_{z_q} and C_{z_δ} are faily well estimated. The inclusion of the second mode (case 2) improves the accuracy of the estimation of parameters C_{z_q} and C_{z_δ} considerably while other parameters are only marginally affected. The matching of true and estimated responses for cases 2 and 3 in Fig. 1 shows that the inclusion of the second mode (case 2) improves matching near the first and the second in-vacuo frequency (around 12–14 rad/s) as compared to case 3. However, the matching near the third and the fourth in-vacuo frequency range (around 21–22 rad/s) is still poor, especially in the phase plot.

For case 4, the parameter extraction algorithm omitted all the aeroelastic effects. The estimated parameters for C2 and C3 configurations are listed in columns 6 and 7 of Table 1. It may be seen that the estimated values for C2 are not far off the true values, but the difference from the true values increases significantly for more flexible C3 configuration. Therefore, the assumption of the rigid body model in estimation algorithm can be only reasonable for nominal aero-

Table 1 Comparison of estimated parameters from estimation algorithm that includes all the modes, first and second modes only, first mode only, and none of the modes

		Estimated parameters					Computed values using Eq. (4)	
Parameters	True values	All modes included (C2)	First and second modes included (C2)	First mode included (C2)	All modes neglected (C2)	All modes neglected (C3)	(C2)	(C3)
$-C_{z_\alpha}$	2.922	2.922 (0)	2.916 (0.21)	2.924 (0.07)	2.719 (7.0)	2.250 (22.9)[a]	2.710	1.995
C_{z_q}	14.700	14.700 (0)	14.86 (1.1)	13.63 (7.2)	15.79 (7.4)	7.77 (47)	16.04	8.79
$-C_{z_\delta}$	0.435	0.435 (0)	0.420 (3.4)	0.579 (33)	0.392 (9.8)	0.059 (86)	0.252	−0.363
$-C_{m_\alpha}$	1.660	1.660 (0)	1.649 (0.67)	1.652 (0.48)	1.421 (14)	0.580 (65)	1.424	0.631
$-C_{m_q}$	34.750	34.750 (0)	34.68 (0.20)	34.77 (0.06)	31.82 (8.4)	26.21 (25)	33.24	28.17
$-C_{m_\delta}$	2.578	2.578 (0)	2.558 (0.78)	2.554 (0.93)	2.349 (8.8)	1.595 (38)	2.375	1.688

[a]Percentage of error with respect to true values.

elastic effects and is quite inadequate to yield any useful estimates as the flexibility of the aircraft increases.

It is of interest to see how the above estimated parameters for case 4 compare with the analytically based values computed using the proposed Eq. (4) for approximating such equivalent estimated parameters that would result from the use of a rigid body model in estimation algorithm. Such computed values for C2 and C3 configurations are shown in columns 8 and 9 of Table 1. Except for $C_{z\delta}$, the agreement in the estimated and computed values is good, with marginal deterioration for configuration C3 as compared to C2.

The above observations suggest that the proposed Eq. (4) may be useful in two different ways.

1) If the rigid body model of a flexible aircraft were tested in a wind tunnel to obtain rigid body stability and control derivatives, and the theoretical values of mode shapes and in-vacuo frequencies of the aeroelastic airplane were calculated, then one could analytically compute from Eq. (4) the expected values of equivalent parameters that would be obtained from flight data of the aeroelastic aircraft if estimation algorithm were to use the rigid body model.

2) Conversely, if rigid body wind tunnel and flight test estimations (based on rigid body model in estimation algorithm) were given, and limited structural data (in-vacuo frequencies and total force coefficients $C_{z_{n1}}$, C_α^{n1}, etc.), then Eq. (4) could be used to compute terms like C_{n1}^{n1}, etc.

The function F defined in Eq. (4) seems promising to quantitatively indicate the degree of flexibility of aircraft and thereby suggest criterion for deciding adequacies or otherwise of using simple rigid body models in estimation algorithms. Work is in progress for evolving such a criterion.

References

[1]Maine, R. E., and Iliff, K. W., "Application of Parameter Estimation to Aircraft Stability and Control," NASA RP 1168, June 1986.

[2]Zerweckh, S. H., Von Flotow, A. H., and Murray, J. E., "Flight Testing a Highly Flexible Aircraft: Case Study on the MIT Light Eagle," *Journal of Aircraft*, Vol. 27, No. 4, 1990, pp. 342–349.

[3]Waszak, M. R., and Schmidt, D. K., "Flight Dynamics of Aeroelastic Vehicle," *Journal of Aircraft*, Vol. 25, No. 6, 1988, pp. 563–571.

[4]Rynaski, E. G., Andrisani, D., II, and Weingarten, N. C., "Identification of the Stability Parameters of an Aeroelastic Airplane," AIAA Paper 78-1328, 1978.

[5]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, 2nd ed., Vol. 1, Addison-Wesley, Reading, MA, 1957, pp. 749–802.

[6]Klein, V., "Estimation of Aircraft Aerodynamic Parameters from Flight Data," *Progress in Aerospace Sciences*, Vol. 26, Vol. 1, 1989, pp. 1–77.

Effects of Model Scale on Flight Characteristics and Design Parameters

William H. Phillips*
NASA Langley Research Center,
Hampton, Virginia 23681

Nomenclature

C_D = drag coefficient
C_L = lift coefficient
c = velocity of sound, or wing chord
D = drag
d = ratio of density, model to full scale
L = length
l = ratio of length, model to full scale
M = mass
m = ratio of mass, model to full scale
P = power
p = ratio of power, model to full scale, or rolling velocity
S = wing area
V = airspeed
W = weight
γ = flight-path angle
μ = viscosity
ρ = density

Subscripts

mod = model
f.s. = full scale

Introduction

FLYING scale models are often used in various phases of aircraft design and research. These models range from subscale prototypes to very small scale spin-tunnel models. While applicable scaling laws are well known to specialists in various fields of research,[1] the effects of scaling on a wide range of parameters involving flight characteristics, performance, and structural design do not appear to be generally

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*Distinguished Research Associate. Fellow AIAA.